

Relativistic interaction of laser pulse with thermal electron plasma

J. Vanchinkhuu

Department of Physics, National University of Mongolia, Ulaanbaatar, Mongolia.

The interaction of a short laser pulse ($\lambda_p \sim L$) with thermal fluid plasma in relativistic case is investigated in hydrodynamical approach. The one dimensional description of this process is developed and the fundamental equations describing the plasma quantities during the interaction are obtained. The wake generation by laser pulse is, also, investigated and the laser wake field in thermal plasma by a laser pulse is handled numerically. In weakly relativistic case, the wake potential propagating in snoidal wave form is obtained. While the wavelength of laser pulse is increased, the wavelength of plasma potential wave in thermal plasma is decreased compared to that in cold plasma. Also, due to the thermal motion, the amplitude of plasma wave is decreased.

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For last decades, the interaction of ultrashort and ultrahigh intensity laser pulse with plasma has become very interesting subject of investigation since it involves many interesting phenomena. On the other hand, this interaction has offered new perspectives of charged particle accelerator [1, 2]. During the laser-plasma interaction, many interesting effects, either in interaction region or behind the laser pulse, such as large amplitude potential plasma wave and wake are observed. Investigation of this interaction is connected to nonlinear relativistic plasma oscillations. The one-dimensional description of relativistic oscillation of thermal electron plasma by electron beams has constructed by T. Katsouleas and W. B. Mori [3] during the investigation of the relativistic wave-breaking process in thermal plasma [4, 5].

In this article, we address the one-dimensional relativistic nonlinear interaction of a short laser pulse with a thermal plasma in the Coulomb gauge. The one-dimensional approach is valid for a laser pulse of a wide front for which the characteristic time of a transverse effect, $\tau = \frac{\pi r_0}{\lambda c}$ (where r_0 is the radius of spot size at the Rayleigh length for the laser pulse), is long to compared to the characteristic time of longitudinal changes, $\tau_l \approx \lambda_p \gamma \frac{n}{n_0} \frac{\lambda_p}{\lambda}$ (where λ and λ_p are the wavelength of the laser pulse and the plasma wave, γ is the relativistic factor). This condition is valid when $r_0 \gg \lambda_p$. We assume that the laser pulse is non-evolving during the interaction because the depletion of pulse is not considerable in the time $t < \left(\frac{8}{3\sqrt{2}}\right) \left(\frac{\omega}{\omega_p}\right) \frac{1}{|\alpha|}$ [6]. This interaction will be considered in fully relativistic manner by using conservation of energy-momentum of fluid. It should be noted that considering of one electron motion in the interaction is almost impossible because of relativistic pressure and internal energy due to the thermal fluid. Although the laser plasma interaction is exactly two-dimensional problem, the dimension of the problem can be reduced for a wide front laser pulse. For sufficiently wide front laser pulse, all laser and plasma quantities are independent of transverse coordinates. Thus, the problem is reduced to one-dimensional problem. This article differs from the

article in ref. [3] by considering the plasma oscillation when laser pulse is present in plasma. Whereas the plasma oscillation is investigated in ref. [4] under the limit, $\frac{\lambda k T}{m_0 c^2} \ll 1$, it is investigated more generally in our case. It is understood from this work that to exactly separate the pressure term ($F_p = -\frac{\gamma^2}{n} \left[\frac{\partial}{\partial x} + \frac{1}{c} \frac{\partial}{\partial t} \right] P$, where $P = \left(\frac{n}{n_0}\right)^3 \frac{kT}{n_0}$ is pressure at low temperature [4]) in the equation of motion is impossible and this is done approximately in the mentioned reference since author replaces $p + \varepsilon$ by $n_0 m c^2$. Its correct form is given by Katsouleas [3].

First of all, we consider the motion of fluid element interacting with nonevolving laser pulse propagating along x . Because of its mass and the shortness of laser pulse the plasma ions are assumed immobile. The motion of fluid element of thermal electron fluid plasma is described by the conservation of energy-momentum, $\frac{\partial T^{ik}}{\partial x^k} = 0$ (T^{ik} is energy momentum tensor of fluid) [3, 7]. For laser pulse and plasma interaction, this holds true but the energy momentum tensor contains the energy-momentum tensor of fluid (T_{fl}^{ik}) and field (T_{fi}^{ik}), $T^{ik} = T_{fl}^{ik} + T_{fi}^{ik}$, and these are determined, respectively, as [8]

$$T_{fi}^{ik} = p g^{ik} + (p + \varepsilon) U^i U^k, \quad (1)$$

$$T_{fl}^{ik} = \frac{1}{4\pi} (-F^{il} F_l^k + \frac{1}{4\pi} g^{ik} F_{lm} F^{lm}), \quad (2)$$

where p and ε are the pressure and the internal energy density of the fluid, respectively, g^{ik} is the metric tensor, $U^i = \left(\frac{1}{\sqrt{1-v^2/c^2}}, \frac{v}{\sqrt{1-v^2/c^2}} \right)$ is the four velocity, $F^{ik} = \frac{\partial A^i}{\partial x^k} - \frac{\partial A^k}{\partial x^i}$ is the electromagnetic field tensor and $A^i = (\varphi, \vec{A})$ is the four vector of the electromagnetic field, φ is the scalar potential, \vec{A} is the vector potential. The conservation of the energy-momentum gives the following equations for its first three components which are independent of transverse coordinates,

$$\frac{1}{c} \frac{\partial}{\partial t} \left(p + \frac{p + \varepsilon}{1 - \beta^2} \right) + \frac{\partial}{\partial x} \left(\frac{p + \varepsilon}{1 - \beta^2} \beta_x \right) = en \left(\beta_x \frac{\partial \varphi}{\partial x} + \frac{\beta_{\perp}}{c} \frac{\partial A_{\perp}}{\partial t} \right), \quad (3)$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{(p + \varepsilon) \beta_x}{1 - \beta^2} \right) + \frac{\partial}{\partial x} \left(-p + \frac{(p + \varepsilon) \beta_x^2}{1 - \beta^2} \right) = en \left(\frac{\partial \varphi}{\partial x} - \frac{\beta_{\perp}}{c} \frac{\partial A_{\perp}}{\partial t} \right), \quad (4)$$

$$\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{(p + \varepsilon) \beta_{\perp}}{1 - \beta^2} \right) + \frac{\partial}{\partial x} \left(\frac{(p + \varepsilon) \beta_x \beta_{\perp}}{1 - \beta^2} \right) = en \left(\frac{\partial A_{\perp}}{\partial t} + \beta_x \frac{\partial A_{\perp}}{\partial x} \right), \quad (5)$$

where e is the electron charge, n is the electron density of plasma, c is the speed of light, $\beta_x = \frac{v_x}{c}$ and $\beta_{\perp} = \frac{v_{\perp}}{c}$ are the normalized longitudinal and transverse velocity of electrons (compared to the propagation direction of the laser), respectively, $\beta = \frac{v}{c} = \sqrt{\left(\frac{v_x}{c}\right)^2 + \left(\frac{v_{\perp}}{c}\right)^2}$ is the total velocity. The first equation implies the last two equations and this corresponds to the equation $\frac{d\varepsilon_k}{dt} = \vec{v} \frac{d\varepsilon_k}{dt} = -e(\vec{v} \cdot \vec{E})$, where ε_k is the kinetic energy. To determine the electron density and potential, one should write the continuity equation and the Poisson's equation

$$\frac{\partial n}{\partial t} + \frac{\partial (nv_x)}{\partial x} = 0 \quad (6)$$

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e(n - n_0) \quad (7)$$

where n_0 is the background ion density of plasma. Eq. (3-7) forms a complete system describing the laser-plasma interaction in nonevolving laser pulse case. From now on, we drop subscript in the vector potential. These equations are written in the Lagrangian coordinate ($\xi = x - wt$, where w is the group velocity), respectively, as

$$\frac{\partial}{\partial \xi} \left[\frac{(p + \varepsilon)(\beta_x - \beta_w)}{1 - \beta^2} - \beta_w p \right] = en \left(\beta_x \frac{\partial \varphi}{\partial \xi} - \beta_w \beta_{\perp} \frac{\partial A}{\partial \xi} \right) \quad (8)$$

$$\frac{\partial}{\partial \xi} \left[\frac{(p + \varepsilon)(\beta_x - \beta_w) \beta_x}{1 - \beta^2} - p \right] = en \left(\frac{\partial \varphi}{\partial \xi} - \beta_{\perp} \frac{\partial A}{\partial \xi} \right) \quad (9)$$

$$\frac{\partial}{\partial \xi} \left[\frac{(p + \varepsilon)(\beta_x - \beta_w) \beta_{\perp}}{1 - \beta^2} \right] = en (\beta_w - \beta_x) \frac{\partial A}{\partial \xi} \quad (10)$$

$$-w \frac{\partial n}{\partial \xi} + \frac{\partial (nv_x)}{\partial \xi} = 0 \quad (11)$$

$$\frac{\partial^2 \varphi}{\partial \xi^2} = 4\pi e(n - n_0) \quad (12)$$

where $\beta_w = \frac{w}{c}$. Recognizing $\frac{n}{n_0} = \frac{\beta_w}{\beta_w - \beta_x}$ from Eq. (11), and dividing Eq. (8) by β_w and subtracting it from Eq. (9), one obtains the following equation for plasma potential.

$$\frac{\partial}{\partial \xi} \left[\frac{(p + \varepsilon)(1 + \beta_x^2 - \beta_x/\beta_w - \beta_x \beta_w)}{1 - \beta^2} \right] = en_0 \frac{\partial \varphi}{\partial \xi} \quad (13)$$

Also, using $\frac{n}{n_0} = \frac{\beta_w}{\beta_w - \beta_x}$ in Eq. (10), one obtains

$$\frac{(p + \varepsilon)(\beta_x - \beta_w) \beta_{\perp}}{1 - \beta^2} = -en_0 \beta_w A. \quad (14)$$

The quantity, p and ε are determined by using the water-bag model [3, 9] as [3]

$$\left\{ \begin{array}{l} \varepsilon \\ p \end{array} \right\} = \frac{m^2 c^3 n_0}{2p_0} \left\{ p' [1 + p'^2]^{1/2} \mp \sinh^{-1}(p') \right\},$$

where $p_0 = \sqrt{3kTm}$, $p' = p_0 \frac{n}{n_0} \sqrt{1 - v^2/c^2}$ thermal momentum of water-bag. Then, the enthalpy, $p + \varepsilon$, is determined as

$$p + \varepsilon = mc^2 n_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta_x/\beta_w} \left(1 + \mu \frac{1 - \beta^2}{(1 - \beta_x/\beta_w)^2} \right)^{1/2}, \quad (15)$$

where $\mu = 3kT/mc^2$, $p_0 = \sqrt{m^2 c^2 \mu}$. Then, from Eq. (13), (14) and Eq. (15), the plasma potential and the transverse velocity are determined, respectively as

$$\phi = \frac{1 - \beta_w \beta_x}{\sqrt{1 - \beta^2}} \left(1 + \mu \frac{1 - \beta^2}{(1 - \beta_x/\beta_w)^2} \right)^{1/2} - \sqrt{1 + \mu}, \quad (16)$$

$$\beta_{\perp} = (1 - \beta_w \beta_x) \frac{a}{\phi + \sqrt{1 + \mu}}, \quad (17)$$

where $\phi = \frac{e}{m c^2} \varphi$ and $a = \frac{e}{m c^2} A$ are the normalized plasma and vector potentials, respectively. The second term in Eq. (16), physically, means potential and so, it is the thermal potential of the plasma. Setting $\mu = 0$ in these expressions gives the corresponding relations in cold plasma case as $\phi = \frac{1 - \beta_w \beta_x}{\sqrt{1 - \beta^2}} - 1$,

$\beta_{\perp} = a\sqrt{1-\beta^2}$. Using Eq. (11), one can write the Poisson's equation as

$$\frac{d^2\phi}{d\xi^2} = k_p^2 \frac{\beta_x}{\beta_w - \beta_x}$$

Combining this with Eq. (16) and Eq. (17), one can find the equation describing the spatial distribution of plasma potential, $\frac{\partial^2\phi}{\partial\xi^2} = k_p^2 G(\mu, a^2, \phi)$, where $G(\mu, a^2, \phi) = \frac{\beta_x(\mu, a^2, \phi)}{\beta_w - \beta_x(\mu, a^2, \phi)}$. This equation is expressed in a very very long form. So, it is not shown here explicitly. This equation is completely nonlinear equation and it is valid for a laser pulse, $-L < \xi < 0$, where L is the length of laser pulse. Also, it implies the wake generation behind the laser pulse. The equation for the wake is the same as the equation describing the relativistic thermal plasma oscillation [3, 4, 10]. Numerical solution of this equation is plotted in Fig. 1 with the longitudinal electric field ($E = -\frac{\partial\phi}{\partial\xi}$) together for a laser pulse of the

form of $a(\xi) = a_0 \sin(\pi\xi/L) \cos(k\xi)$, (where $k = \frac{2\pi}{\lambda}$, λ is the wavelength of the laser pulse) with Cauchy condition, $\phi(0) = 0$, $\phi'(0) = 0$ at $\beta_w = 0.95c$. This shows that the contribution of thermal electron in plasma wave is very small. The wavelength and the amplitude of the plasma wave is slightly decreased compared to the cold case. But, it is considerable for a weak intense, long ($L > \lambda_p$) laser pulse and it is apparent in Fig. 2. The wavelength of the plasma wave is decreased due to the thermal motion. As increasing of the intensity of the driving laser pulse, values of the minimum potential and maximum electric field tend to coincide at one coordinate. The amplitude of the plasma potential depends strongly on the length and intensity of laser pulse. It should be noted that the amplitude and wave length of wake potential is increased enormously, when the value of potential at the end of the pulse approaches to $\phi = -\sqrt{1+\mu}$. It is noticed that the longitudinal velocity β_x in the numerical calculation above is found from the following equation:

$$\Phi^4 \delta_x^2 / \gamma_x - \Phi^2 (1 - \beta_x \beta_w)^2 [\delta_x^2 a_0 + \mu / \gamma_x] + \mu (1 - \beta_x \beta_w)^4 a^2 = 0,$$

where $\Phi = \phi + \sqrt{1+\mu}$ is an effective potential, $\gamma_x = (1 - \beta_x^2)^{-1}$, $\delta_x = 1 - \beta_x / \beta_w$, $a_0 = 1 + a^2$. When $\beta_w = 1$, the longitudinal velocity is obtained as:

$$\beta_x = \frac{(a_0 - \Phi^2 + \mu) - \mu a^2}{(a_0 + \Phi^2 + \mu) - \mu a^2}. \quad (18)$$

In this case, the Poisson's equation takes the following form:

$$\frac{d^2\Phi}{d\xi^2} = k_p^2 \left[\frac{a_0}{2(\Phi^2 - \mu)} - \frac{\mu a^2}{2\Phi^2(\Phi^2 - \mu)} - \frac{1}{2} \right]. \quad (19)$$

One can obtain the corresponding longitudinal velocity and the Poisson's equation for the cold plasma ($T \approx 0$, consequently, $\mu \approx 0$ for a cold plasma) from Eq. (18) and (19).

We consider this interaction in weakly relativistic case in which the transverse velocity of electrons by the electric field of the laser pulse is many times higher than the longitudinal velocity, ($\beta_{\perp} \gg \beta_{\parallel}$) because of strong electric field of the pulse [2]. In this case, we can limit the longitudinal velocity by its first order. Under this assumption, one can find the longitudinal velocity, from Eq. (16) and (17), as

$$\beta_x = \frac{\beta_w}{2} \frac{\Phi^4 - (\mu + a_0)\Phi^2 + \mu a^2}{\Phi^4 - (\beta_w^2(\mu + a_0) + a_0)\Phi^2 + 4\mu a^2 \beta_w^2}, \quad (20)$$

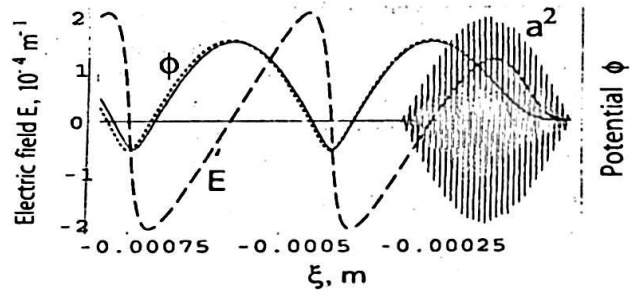


FIG. 1: Potential (the solid line is for a thermal plasma, $T = 5.5 \times 10^7$ K, short dashed line is for cold case) and electric field (long dashed line) excited by short laser pulse ($L = 0.0003$ m).

and correspondingly the Poisson's equation takes the following form:

$$\frac{d^2\Phi}{d\xi^2} = \frac{k_p^2(\Phi^4 - \chi\Phi^2 + \mu a^2)}{\Phi^4 - ((2\beta_w^2 - 1)\chi + 2a_0)\Phi^2 + \mu a^2 \omega} \quad (21)$$

where $\chi = \mu + a_0$, $\omega = 4\beta_w^2 - 1$. This equation with appropriate initial conditions, $\Phi(0) = \sqrt{1+\mu}$, $\Phi'(0) = 0$ gives the plasma wake potential in weakly relativistic case when $a^2 = 0$. The numerical solution

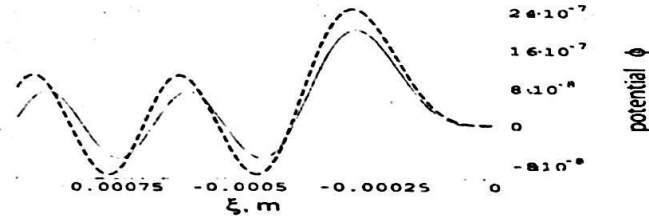


FIG. 2: Wake potential excited by a sine type laser pulse $a_0 = 8 \times 10^{-4}$ ($L = 0.00045$ m) in cold (dashed-line) and thermal plasma, $T = 8.4 \times 10^8$ K (solid line). The weakly relativistic solution is shown by dotted line.

of this equation is shown in Fig. 2 and no considerable difference from the exact solution is observed. This shows that the weakly relativistic approxima-

tion is fully acceptable in the case considered. The lowest limit of velocity is determined from Eq. (20) as

$$\beta_{z \min} \geq \frac{\beta_w}{2} \frac{1 + \mu}{(1 + \mu)\beta_w^2 + 1}.$$

When $a^2 = \text{const}$, the order of Eq. (21) can be reduced by using the identity $2 \frac{dy}{dx} \frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{\partial y}{\partial x} \right]^2$. This situation is valid for a square-shaped laser pulse. However, we consider only the wake potential. Changing the length of laser pulse, one can choose the optimal length of laser pulse that gives the minimum value of the plasma potential, Φ_0 , at its end. At the point corresponding to the minimum potential, the electric field becomes zero. In this case, the electric field of wake is

$$\vec{E} = -\frac{d\Phi}{d\xi} = -k_p \sqrt{2(\Phi - \Phi_0) + \frac{2[1 - \mu_0(1 - \beta_w^2)]}{\mu'} \ln \frac{\mu' + \Phi_0 \mu' - \Phi}{\mu' - \Phi_0 \mu' + \Phi}},$$

where $\mu' = \sqrt{(1 + \mu)(2\beta_w^2 - 1) + 2}$, $\mu_0 = 1 + \mu$. Here, we have used the initial condition, $\Phi(-L) = \Phi_0$, and $\Phi'(-L) = 0$. The plasma potential corresponding to the wavebreaking in wake ($a^2 = 0$), at which the electrons move with the group velocity of wave and the density becomes singular, is determined from Eq. (20) as $\Phi_{wb} = \sqrt{(2\beta_w^2 - 1)\mu_0 + 2}$

[11]. This is the smallest value of the potential that it can reach in weakly relativistic case. Since the minimum value of the potential never reaches to this value as increasing the wave amplitude, we should take it as $\Phi_{\min} = \sqrt{(2\beta_w^2 - 1)\mu_0 + 2}/\sqrt{1 - \beta_w^2}$. Corresponding to this, the maximum value of electric field behind the pulse is determined to be

$$E_{\max}^2 = \frac{m^2 \omega_p^2 c^2}{e^2} \left[2(\gamma_w \Phi_{\min} - \Phi_0) - \frac{2[1 - \mu_0/\gamma_w]}{\Phi_{\min}} \ln \frac{1 + \Phi_0/\Phi_{\min} - 1/\gamma_w}{1 - \Phi_0/\Phi_{\min} - 1/\gamma_w} \right],$$

where $\gamma_w = \frac{1}{\sqrt{1 - \beta_w^2}}$. This tells us that the highest value of the electric field is always smaller than the wavebreaking field in cold plasma, $E_{wb} = \sqrt{\frac{2}{\sqrt{1 - \beta_w^2}} - 2}$ and it depends on the initial intensity of the excitation.

We can choose the length of the laser pulse to obtain an oscillation of small amplitude of potential ($\Phi^n \ll 1$, $n \leq 2$ a number) in the wake. At the point corresponding to the maximum potential, the electric field becomes zero. In this case, we can ex-

press the wake potential analytically. The weakly relativistic wake potential of small amplitude is the solution of Eq. (21) when $a^2 = 0$ and $\Phi^n \ll 1$, $n \leq 2$,

$$\frac{d^2\Phi}{d\xi^2} = k_p^2 \left[\frac{\mu_0}{\mu'^2} - \frac{2[1 - \mu_0(1 - \beta_w^2)]}{\mu'^4} \Phi^2 \right].$$

The solution of this equation is [13]

$$\Phi(\xi) = \sqrt{\frac{3\mu_0[(1-2\beta_w^2)\mu_0+2]}{2[1-\mu_0(1-\beta^2)]}} \operatorname{sn}^2\left[\sqrt{\frac{1[1-\mu_0(1-\beta^2)]\mu_0}{6[(1-2\beta_w^2)\mu_0+2]}} \xi, m\right],$$

where $\operatorname{sn}(u)$ is the sine amplitude the Jacobian elliptic function. This shows that when the amplitude of plasma potential (or amplitude of the laser pulse) is small, it performs snoidal oscillation or it propagates in soliton form. As usual, its period is $4K(m = -1, \pi/2)$ ($K(m, \pi/2)$ is the complete elliptic integral of first kind) and is independent of temperature.

Up to now, we have studied the behavior of plasma for a nonevolving laser pulse. In fact, the effects in interacting channel affect the laser pulse, so it is changed during the interaction. At this stage, consider the behavior the laser pulse. The propagation of laser pulse is governed by the wave equation in Coulomb gauge. It is written for the normalized vector potential in moving frame ($\xi = x - wt, \tau = t$)

with the group velocity, $v_g = w = \frac{c^2 k_0}{\omega_0}$ (where ω_0 and k_0 are the initial frequency and wave number of the laser pulse) as:

$$(1 - \beta_w^2) \frac{\partial^2 a}{\partial \xi^2} - \frac{1}{c^2} \frac{\partial^2 a}{\partial \tau^2} + \frac{2\beta_w}{c} \frac{\partial^2 a}{\partial \xi \partial \tau} = k_p^2 \frac{n}{n_0} \beta_{\perp}.$$

Recalling the linear dispersion relation, $\omega_0^2 = k_0^2 c^2 + \omega_p^2$, one can find that $(1 - \beta_w^2) = \frac{\omega_p^2}{\omega_0^2}$. Introducing the laser pulse $a(\xi, \tau) = a_0(\xi, \tau) \exp(ik\xi) + c.c.$ in the equation above with Eq. (16) and the continuity equation, one can find the following relation for k and ω (ω and k are the frequency and wave number of the laser pulse)

$$-\frac{\omega_p^2 k^2}{\omega_0^2} + k^2 + \frac{\omega^2}{c^2} - \frac{2k\omega}{c} - \frac{2k_0}{\omega_0} ck^2 + \frac{2k_0}{\omega_0} k\omega = k_p^2 \frac{1 - \beta_w \beta_x}{1 - \beta_x / \beta_w} \frac{1}{\phi + \sqrt{\mu_0}}.$$

Here, we have replaced $\frac{\partial}{\partial \xi}$ with ik and $\frac{\partial}{\partial \tau}$ with $i(c k -$

$\omega)$ [12]. By using Eq. (20), it is written as

$$k^2 \left(1 - \frac{\omega_p^2}{\omega_0^2} - \frac{2k_0}{\omega_0 c} \right) + \frac{\omega^2}{c^2} - 2k\omega \left(\frac{1}{c} - \frac{k_0}{\omega_0} \right) = \frac{k_p^2}{\phi + \sqrt{\mu_0}} \frac{\Phi^4(2 - \beta_w^2) - \Phi^2[\mu_a \beta_w^2 + 2a_0]}{\Phi^4 - (\mu_a(2\beta_w^2 - 1) + 2a_0) \Phi^2 - \mu_a^2(1 + 2\beta_w - 2\beta_w^2)}$$

where $\mu_a = \mu_0 + a^2$. With the solution of Eq. (21), it yields the dispersion relation of the thermal electron plasma for a laser pulse. For a long laser pulse, the plasma potential in the weak density limit is expressed by the intensity of the laser pulse, from Eq. (19) as $\Phi = \sqrt{\mu_a(1 + \sqrt{1 - \mu_a^2/\mu_0})}/2$. In the frame moving with speed of light (setting $w = c$ or $\frac{\omega_0}{k_0} = c$), it is

$$\frac{\omega^2}{c^2} = k^2 + \frac{k_p^2}{\sqrt{\mu_a(1 + \sqrt{1 - \mu_a^2/\mu_0})}/2 - \mu}$$

This differs from the well-known dispersion relation of cold plasma [1] by only thermal potential. This tells us that whereas the thermal motion of electrons

makes the laser wavelength lengthened, the plasma wavelength is shortened by thermal motion. Thus, for thermal plasma, the Rayleigh length is short and the group velocity of the plasma wave is fast compared to the cold one. This is caused purely by thermal motion of electrons. Seemingly, the wake generation in thermal plasma is almost as if it is in cold plasma. In another words, the impact of temperature on plasma quantities such as potential and electron density, is negligible in small temperature range. Thermal effects dominate in the interaction processes when the thermal energy is comparable to rest energy of electron which is approximately $mc^2 \sim 10^9$ K. This is too high to be fulfilled in laboratory conditions. But, this does not mean that ther-

mal plasma is not benefited in wake field accelerator effectively. For a relatively hot plasma, it is possible to generate a plasma wave, the amplitude of which is comparable to that in cold plasma. The large amplitude plasma wave generated in thermal plasma by a laser pulse can be used to accelerate a charged particle. But, it is seen that the LWFA (Laser Wake Field Accelerator) in thermal plasma has some disadvantages compared to the LWFA in cold plasma. First of all, the wavelength of plasma wave is shortened by thermal electron motion. Correspondingly, the group velocity is increased and so, the dephasing [14] or detuning length of electron in the wave

becomes shorter than that in a cold plasma. Since the Rayleigh length of the laser in thermal plasma is short compared to that in a cold plasma, the depletion length of laser pulse is also shortened. Finally, it is concluded that the laser and plasma quantities in LWFA in thermal plasma are reduced by the pressure.

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